DYNAMICS, GEODESY AND NAVIGATION IN THE SOLAR SYSTEM

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On-going space missions for planetary exploration (Cassini, Rosetta, etc.) and those which are under study (BepiColombo, JIMO, Europa Orbiter, etc) offer new and appealing opportunities for the study of the physics of the Solar System. As a major advance with respect to past space missions, which were based on single flybys of one or more bodies, future missions will place a spacecraft in orbit around individual bodies of the Solar System (a planet or a planetary moon). This will result in a long observation of the primary body and its satellite system thus enabling the study of unobserved dynamical and evolutionary effects, such as the precise rotational motion of the satellites and their internal structure. As a matter of fact, together with the improvement of orbital tracking precision (connected with the Ka band at around 32 GHz), the times are mature for the extension of the high precision methods developed in the field of Space and Satellite Geodesy, and up to now applied only in terrestrial studies (Lageos I and II, TOPEX, GPS; Champ, GRACE), to the entire Solar System. Such extension and application of the techniques of Space Geodesy to the dynamics and navigation in the Solar System have represented the objective of the research programme of our group for the years 2000-2004 as well as for the foreseeable future. The specific projects in which this programme is divided are presented and discussed in dedicated sections below.

Project 1: *Cassini orbit determination*.
Our group is involved in the Italian Cassini Radio Science team on a project that aims at the determination of the moments of inertia of Titan through radio and radar data. The scientific objective is understanding the internal structure and composition of Titan, which is believed to have a heavy core. The adopted investigation method is the measurement of the perturbations on the spacecraft orbit caused by the quadrupole moments of Titan and the direct measurement of Titan's obliquity via SAR imaging. The fundamental requirement for the determination of the inertia moments with an accuracy of 5% is the reconstruction of Cassini's orbit with an accuracy of 300 meters and the establishment of a geodetic network on the surface of Titan of comparable accuracy. This study will benefit from the theoretical analysis of the rotational dynamics of the natural satellites in the Solar System and in particular from the analytical study of the states of spin-orbit coupling. The results will find application also to studies of the planet Mercury.

Project 2: *Development of the gravitational perturbation spectrum of a body orbiting in a multi-satellite system of natural bodies moving along prescribed trajectories*.
The method is a completely analytical approach to transforming the theories of the motion of the satellites into the coefficients of the tidal potential of the satellite system. The method is an extension of the one used to reproduce and extend Doodson's celebrated 1921 development for the Sun-Earth-Moon system using the rotation and translation theorems for spherical harmonic functions. This multi-body tidal potential development can be used for a quick assessment of the perturbation spectrum affecting the motion of a particle orbiting, or passing through the multi-satellite system or in computing the torques determining the rotational motion of bodies in the system (a prerequisite for the development of their precession and nutation series). It can also be used for geophysical or geodetic applications to system bodies, like the tidal displacement of their surfaces. In particular, the development has been carried out for the Saturnian satellite system because of the current interest in the Cassini mission, although applications of the present methodology to other satellite systems in the Solar System are also possible. A possible benefit of the present development is in the analysis of the motion of newly-discovered small moons and to the early planning phases of space missions.

Project 3: *Mission design*. In this field our group is performing a systematic classification of the various types of orbital maneouvres for interplanetary missions with an aim at the application to optimization methods based on parameter estimation.
With reference to the Mars exploration programme, strongly supported by the Italian Space Agency for its scientific and technological implications, we have investigated and proposed the

observation of the inner satellite Phobos as a phase of a more general Martian mission. The concept is that the surface of Phobos can be scanned from a polar, arecentric orbit, slightly manoeuvred at twice the orbital frequency to achieve total coverage by performing two close encounters on the sunlit side of the satellite per orbital period. As far as trajectory optimization is concerned, we are studying a novel approach to the design of low-thrust transfers, based on dynamic programming.
The optimal control problem associated with low-thrust trajectories is solved through a backward stage-wise local solution of the Hamilton-Jacobi-Bellman equation. The whole trajectory is decomposed into a sequence of single stage problems of small dimensions, which are iteratively solved backward in time yielding the optimal control profile. This leads to a quadratic convergent algorithm whose computational cost grows linearly with the number of stages. This approach is more robust than traditional indirect methods and, for a high number of stages, more efficient than direct approaches. Moreover the stage-wise solution of the problem allows, at each optimization step, an adjustment of the discretization grid through a variable step integration scheme. This property is particularly appealing in the treatment of variable scale highly non linear dynamics.
Further studies concern the development of tools for preliminary trajectory design based on global optimization with genetic algorithms and simplified models of trajectory transcription.

Progetto 4: *Earth's Gravity field determination: satellite gravity gradiometry, orbit determination*.
Terrestrial gravity field models are spherical harmonics series expansions in terms of the Stokes coefficients that represent the inhomogeneous mass distribution within the Earth. Knowledge of the gravity field is essential to computation of artificial satellite orbits, geodesy, positioning and navigation, Earth's rotation, oceanography and study of the climate. Geopotential models are obtained by analysing the perturbations affecting artificial satellite orbits, in combination with altimetry and gravimetry data. Detailed comparisons among several models indicate lack of accuracy due to geographical inhomogeneities (large unobserved areas) and limited degree and order of the series expansion.
Recently, new efforts have been made to improve our knowledge of the geopotential. The idea consists of satellite measurements of functionals that are more sensitive to the structure of the field. In particular, three research lines have developed, each of which corresponds to a space mission: CHAMP, GRACE and GOCE.
The processing of GPS tracking and accelerometry data for the CHAMP satellite has produced two models, EIGEN–1S and EIGEN –2 that reach degree and order 120, a considerable improvement if compared to the maximum degree and order 70 of the previous models. The processing of GRACE data, based on the concept of microwave dual tracking between two satellites at a relative distance of about 100 km, aims at mapping the temporal variations of the field.
The planned GOCE mission, which is characterized by an important Italian participation, is based on the principle of satellite gravity gradiometry (SGG) in combination with satellite-to-satellite tracking (SST-hl). The two techniques complement each other: the differential SGG measurement is very sensitive to high spatial frequencies and quite insensitive to low frequencies; SST, on the contrary, is capable of accurately providing the low frequencies of the gravity field because these result from a double integration of the equations of motion.
Our contribution to the precise orbit determination of GOCE involves the use and upgrade of the Padua Orbit Propagator (POP) software. A more accurate force model is required to appropriately deal with the low orbit (250 km) of GOCE and with the strong interactions between the satellite and the Earth's atmosphere. The necessary changes and upgrades are being studied and implemented thanks to the GRACE and CHAMP data, in collaboration with the Center for Space Research of the University of Texas at Austin and GFZ (Potsdam).
As for SGG, we are currently investigating data reduction strategies to estimate the Stokes geopotential coefficients from the raw data (the gravity gradients measured by the 6 accelerometers onboard the spacecraft). Since the observation of gravity gradients by means of an electrostatic gradiometer onboard a satellite like GOCE is a completely unexplored technique, the research and the formulation of robust algorithms for data inversion currently constitutes an issue of primary importance.

Given the high density measurements (1 second sampling interval) and the duration of the mission (2 observation periods of 6 months each), the inversion problem is a giant numerical challenge, further complicated by the characteristics of the measurement noise.
Important findings of our work concern the choice of the method to simulate the gravity gradients (synthesis: a fundamental phase in any iterative solution procedure): a thorough comparison among various approaches has shown that Clenshaw method guarantees the lowest computational cost.
On a more theoretical side our research group is investigating the use of spheoridal harmonics as an alternative method to represent the gravity field of irregular bodies. Progress has been made on the computational side of the special functions involved and numerical tests are underway in applications to test cases.

Project 5: We have recently started investigations in the field of transfer trajectory design in multi-body systems on the basis of Dynamical Systems Theory. The main advantage of these techniques is that they exploit the dynamical characteristics of multi-body systems to identify transfer trajectories requiring very low energy input, which would otherwise go unobserved if classical two-body transfer techniques were applied. The hyperbolic invariant manifolds associated with periodic orbits about libration points have proven to be a key ingredient in low-energy trajectory design and transport dynamics in the Solar System. Much research has been carried out in this area based on the paradigmatic case of the Circular Restricted Three-Body Problem. Many astrophysical systems, however, exhibit primaries in highly eccentric motion, thus requiring the adoption of the Elliptic Restricted Three-Body Problem (ERTBP) as a more appropriate reference model to accurately describe the dynamics of natural and possibly artificial bodies within them. The dynamical characteristics of the ERTBP, notably the absence of the Jacobi integral, the explicit time-dependence of the equations of motion in the synodical frame, the non-degeneracy of the spectrum of the periods of the periodic orbits, and the characteristics of the eigenvalues of the monodromy matrix, make it very different from its circular analogue. We have used Dynamical Systems Theory and associated numerical techniques to explore families of periodic orbits and their invariant manifolds aiming at the design of heteroclinic, low-energy transfer orbits in the framework of the Elliptic Restricted Three-Body Problem.

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Research Products
2000:

1 refereed papers
3 conference proceedings
2001:
3 conference proceedings
2002:
4 conference proceedings
2003:
2 conference proceedings
2004:
1 refereed papers
6 conference proceedings
1 technical report

Tutorship activity and thesis supervisions:

- Tesi di Laurea. Title: Progettazione di orbite di trasferimento planetocentriche ed interplanetarie. Candidate: Sara Poltronieri. Academic year: 2003/2004
- Tesi di Laurea. Title: Sulla rappresentazione del potenziale gravitazionale in armoniche sferiche e sferoidali. Candidate: Patrizia Borin. Academic year: 2003/2004.
- Tesi di Laurea. Title: Orbite a bassa energia nel problema dei tre corpi ellittico ristretto Candidate: Elisa Maria Alessi. Academic year: 2003/2004
- Tesi di Laurea. Title: Sviluppo analitico del Potenziale Generatore di Marea nel sistema Sole-Terra-Luna. Candidate: Francesco Biscani. Academic year: 2003/2004.
- Tesi di Laurea. Title: Studio di missione per imaging ad alta risoluzione di Phobos. Candidate: Alessandro Tonello. Academic year: 2003/2004
- Tesi di Laurea. Title: Determinazione d'orbita per una sonda interplanetaria mediante misure Doppler. Candidate: Andrea Nardo. Academic year: 2003/2004.
- Tesi di Laurea. Title: Determinazione dei parametri di monopolo e quadrupolo di Titano nella missione Cassini. Candidate: Massimo Bardella. Academic year 2000/2001.
- Tesi di Laurea. Title: Posizionamento puntuale di piattaforme statiche e orbitanti mediante il global positioning system. Candidate: Boris Padovan. Academic year: 2000/2001.

PhD Thesis (31/12/2001): Precise orbit determination of low-Earth orbiting satellites using the Global Navigation Satellite System (GNSS). Candidato: Alberto Zin.

List of 5 most representative publications:

- FANTINO E., CASOTTO S. (2004). *Study on the libration points of the Sun and the interstellar medium for interstellar travel*. http://www.esa.int/gsp/ACT/ariadna/completed_studies.htm. Report prepared for ESA's Advanced Concepts Team (ACT).
- CASOTTO S., BISCANI F. (2004). *A fully analytical approach to the harmonic development of the tide-generating potential accounting for precession, nutation, and perturbations due to figure and planetary terms*. AAS/Division of Dynamical Astronomy Meeting. (vol. 35).
- DE PASCALE P., VASILE M., CASOTTO S. (2004). *Preliminary Analysis of Low-Thrust Gravity Assist Trajectories by An Inverse Method and a Global Optimization Technique*. International Symposium on Space Flight Dynamics. 11-15 Oct. 2004, Munich (Germany).
- CASOTTO S., TONELLO A., BARDELLA M. (2004). *Phobos Imaging and Mapping Preliminary Mission Design*. American Astronautical Society Space Flight Mechanics Meeting. (vol. 119). Advances in the Astronautical Sciences (AAS 04-233).
- PELLETIER F.J., CASOTTO S., ZIN A., PADOVAN B. (2003). *Preliminary Assessment of Interferometric SAR Baseline Determination Using GPS*. American Astronautical Society Spaceflight Mechanics Conference. (vol. 114, pp. 561-572). ISBN/ISSN: 0-87703-504-0.