4. GALAXY INTRACLUSTER PLASMAS

In the current Section we apply all concepts and formulas on hydrodynamics, plasma physics and free-free emission processes presented in Sects. 2 and 3, to analyze the properties of hot plasmas in clusters of galaxies.

These latter make a fundamental astrophysical component, including a large fraction of cosmic baryons, and it is then of relevance not only for High-Energy Astrophysics, but for cosmology and galaxy/stellar astrophysics too.

4.1 The fundamental physical parameters

4.1.1 Particle mean free paths.

From eq. (3.56) we have

$$\lambda_e \simeq \frac{7 \times 10^5}{N \ln \Lambda} T^2 \quad [cm] \simeq \lambda_i$$

corresponding to

$$\lambda_e = \lambda_i \approx 23 \, Kpc \left(\frac{T}{10^8 \, K}\right)^2 \left(\frac{n_e}{10^{-3} \, cm^{-3}}\right)^{-1}$$

having assumed the same thermodynamic temperature for electrons and protons, $T_e \approx T_{ion}$. These mean free paths are then much lower than the typical spatial scales of interest in clusters (>1 Mpc), so that the Intra-Cluster Plasma can be treated as a collisional fluid following the laws of hydro-dynamics.

However, $\lambda$ has the typical size of a galaxy, and the interactions galaxy-plasma can be non-collisional. In addition, if galaxies contribute energy to the IC plasma via galactic winds or non-thermal processes, we can expect some effects of non-homogenization or isotropization in the vicinity of galaxies.

4.1.2 Thermalization timescales.

The timescales for thermalization are immediately calculated from our analysis in Sect. 3.7. We have from eq. (59)
\[ t_{E}(e-e) = \frac{m_e^{1/2}(kT)^{3/2}}{Ne^4 \ln \Lambda} \approx 3.3 \times 10^5 \text{ yrs} \left( \frac{T_e}{10^8 \text{ K}} \right)^{3/2} \left( \frac{n_e}{10^{-3} \text{ cm}^{-3}} \right)^{-1} \]

\[ t_{E}(i-i) = \frac{m_i^{1/2}(kT)^{3/2}}{Ne^4 \ln \Lambda} \approx 43 \times t_{E}(e-e) \]

\[ t_{E}(e-i) = 1840 \times t_{E}(e-e) = 6 \times 10^8 \text{ yrs} \left( \frac{T_e}{10^8 \text{ K}} \right)^{3/2} \left( \frac{n_e}{10^{-3} \text{ cm}^{-3}} \right)^{-1} \]

de the latter due to the fact that for every collision only a fraction \( \approx 1/1840 \) of the energy is exchanged.

So in the IC plasma, the longest times for achieving equilibrium is that between electrons and ions \( t_{E}(e-i) \approx 10^9 \text{ yrs} \). Then if there might be heating sources currently active in the cluster, we might expect significant differences in the electron and ion temperatures \( T_e \) and \( T_i \) that would manifest, for example, in the ionization collisional equilibrium in the plasma and testable via the emission line analysis (see Sect. 4.2).

However, note that such longest times are in any case much shorter than the age of the cluster and of the plasma cooling time (Sect. 2.7.3). So with good approximation, the plasma can be considered as fully thermalized at a unique temperature \( T \approx T_e = T_i \).

### 4.1.3 Thermal conduction.

From eq. (3.72-74) we have that the heat flow following a gradient of temperature is

\[ \bar{q} = -k_c \bar{\nabla} T_e \]  

with

\[ k_c = 4.6 \times 10^{13} \left( \frac{T}{10^8 \text{ K}} \right)^{5/2} \left( \frac{\ln \Lambda}{40} \right)^{-1} \text{ [erg/sec/cm/K]} \]

The latter is the thermal conductivity of the plasma (Spitzer 1956). From eq. (3.6) of the energy balance in the fluid, assuming a stationary (v=0) hydrostatic and adiabatic condition (cooling timescale for radiation very long compared to the cooling for conduction, or \( F_{rad} = 0 \)), we have

\[ \frac{\partial}{\partial t} (\rho \varepsilon) = -\bar{\nabla} \cdot \bar{q}, \quad \varepsilon = \frac{3}{2} \frac{kT}{\mu m_H} \]
Assuming that the plasma density should not vary much with time, we have the condition

\[
\frac{3}{2} \frac{\rho k}{\mu m_{H}} \frac{dI_g}{dt} = -\nabla \cdot \vec{q}
\]

where \( T_g \) is the gas temperature. The characteristic time-scale for heat conduction in the plasma can be defined as

\[
t_{\text{cond}} = T_g \left( \frac{dI_g}{dt} \right)^{\frac{1}{2}}
\]

This quantity can be calculated from assuming a King profile (see Sect. 4.3.3 below) for the gravitational potential and gas density distribution. The result is reported in Figure 1, where the cooling time is reported normalized to the value at the cluster center.

![Figure 1](image)

The conduction time as a function of position in an adiabatic model for the intracluster gas. The conduction time is relative to its central value, and the radius is in units of the cluster core radius \( r_c \). The solid (dashed) curve indicates the portion of the cluster where the gas is cooled (heated) by conduction.

At the center of the cluster, we calculate the characteristic time-scale for conduction as follows. From (6) we have

\[
\frac{dT_g}{dt} = \hat{\nabla} \cdot \hat{q} \quad \text{with} \quad \hat{q} = -k_c \hat{\nabla} T_g \approx \frac{k_c T_g}{l_T^2}
\]

where we have introduced a characteristic length for the variation of temperature.
\[ l_T \equiv \frac{T_g}{\nabla T_g} \]

Then, assuming that the length-scale \( l_T \) is of the order of the core radius of the cluster, \( l_T \approx r_C \), we have

\[ t_{\text{cond}} \equiv T_g \left( \frac{dT_g}{dt} \right)^{-1} = \frac{T_g}{1} \frac{k_c T_g}{k_c} \frac{n_0 l_T^2 k}{n_0 k \lambda T} \approx \frac{n_0 r_C^2 k}{k_C} \]

\[ \approx 3 \times 10^8 \, \text{yrs} \left( \frac{n_0}{10^{-3} \, \text{cm}^{-3}} \right) \left( \frac{T_g}{10^8} \right)^{-5/2} \left( \frac{r_C}{0.25 \, \text{Mpc}} \right)^2 \]

with \( n_0 = \rho / \mu m_H \) the plasma number density.

We see from this that heat conduction is efficient in the cluster center, but becomes quickly inefficient outside the cluster core radius \( r_C \). In the figure, the branch at \( r < 2.5 \, r_C \) corresponds to cooling of the plasma, \( r > 2.5 \, r_C \) it corresponds to heating by the hot inner cluster plasma.

### 4.1.4 Effects of the presence of an intra-cluster magnetic field

A further limitation to the thermal conduction in the cluster is due to the presence of a magnetic field \( \vec{B} \), as discussed in Sect. 3 chapt. 12. This will tend to decrease the effectiveness of the conduction and to increase the conduction timescales \( t_{\text{cond}} \) with respect to what previously discussed. From eq. (3.100), adopting the characteristic values of IC plasmas,

\[ r_g \approx \frac{3 \times 10^8 \, \text{cm}}{Z} \left( \frac{T_g}{10^8} \right)^{1/2} \left( \frac{m}{m_e} \right)^{1/2} \left( \frac{B}{1 \, \mu \text{Gauss}} \right)^{-1} \]

that is much less than any relevant scale-length in the IC plasma and much less than the mean free path of electrons, \( r_g \ll \lambda_e \). Observations of \( B \) fields in clusters will be discussed in Sect. 10 based on the Faraday rotation effect.

Because they have much larger gyration radii, ions are more effective in transporting energy perpendicularly to \( \vec{B} \). In practice, orthogonally to \( \vec{B} \) the diffusion and conduction are virtually suppressed.

Note however that this depends on the spatial structure of the magnetic field. How is magnetic field structured inside the IC plasmas? The observations rely on the
Faraday rotation effect (as discussed in Sect. 10). These show quite entangled magnetic field lines, over scale-lengths

\[ \lambda_B = \lambda_e = 20 \text{ kpc} \]

where \( \lambda_B \) is the scale over which the magnetic field direction changes (\( \tilde{B} \) changes by 90 degrees on average after a distance equal to \( \lambda_B \)). We can imagine here three cases to apply.

- Case with \( \lambda_B > \lambda_T >> \lambda_e \) (unsaturated conduction). In this case the \( \tilde{B} \) field is well spatially ordered, in which case there is a factor \( \cos^2 \vartheta \) of reduction in the heat flow, \( \vartheta \) being the angle between the temperature gradient and the direction of \( \tilde{B} \).

- Case with \( \lambda_T >> \lambda_B > \lambda_e \). In this case the direction of \( \tilde{B} \) is a random variable, producing an attenuation of the heat flow by \( \langle \cos^2 \vartheta \rangle = 1/3 \).

- Case with \( \lambda_B << \lambda_e \). The scale-length of diffusion is \( \lambda_B \). In this case we expect a drastic reduction of the conduction (even the microscopic Brownian motions are damped).

In our case we are in a condition at the boundary between the former two.

Therefore we are at the limit of expecting some influence by the entangled magnetic field on the thermodynamical structure of the plasma and some limitation to thermal conduction.

Observationally, we report in the figure a few data about the temperature distributions in the IC plasmas of a few clusters and groups. Some other higher quality data are reported in the figure to the right.

---

1 There are two \( \cos \vartheta \) factors here. This can be seen from eq. (8) for example, where the heat flow is proportional to \( l_T^{-2} \). Each one of these is increased by a factor \( \cos \vartheta \).
Figure 3. Scaled temperature profiles of galaxy clusters derived from spectroscopic observations with the Chandra satellite (left, Vikhlinin et al. 2006) and results obtained for a sample of clusters observed with XMM-Newton (right, Pratt et al. 2007).
Ionization Equilibrium in the Hot IC Plasma

4.2.1 Ionization equilibrium

We have seen that, generally speaking, the dynamical processes in clusters are sufficiently effective, the time-scales for relaxation of the various components are short enough, and the temperature gradients sufficiently moderate that we can assume that over large chunks of the cluster volume the plasma has a single average temperature $T_k$. This temperature may vary smoothly as a function of the cluster position.

The radiation field is low enough that the processes of ionization and excitation by radiative exchanges can be neglected compared to the collisional ones due to particle interactions.

So the ionization status of the plasma is dominated by the numerous collisions between ions and electrons. The latter are essentially 2-body encounters (3-body encounters are very unlikely).

We can further assume that the time-scales for ionization, excitation and recombination are short compared to any other characteristic timescale. → This implies that we can assume an ionization equilibrium to apply in our plasma.

The plasma of the ICM is very tenuous, with densities of $10^{-2} - 10^{-5}$ cm$^{-3}$ from the center of the cluster to the outskirts. This low plasma density makes the cluster X-ray spectra modeling simple and enables a very straightforward interpretation. There are three fundamental emission processes involving electronic “transitions” that contribute to the radiation. Free–free or bremsstrahlung radiation caused by the deflection of an electron at close fly-by of an ion, free–bound or recombination radiation caused by the capture of an electron by an ion reducing the ionization degree, and bound–bound or de-excitation radiation of an electron changing the quantum level in an atom. The first two processes give rise to continuum radiation and the latter to line radiation.

All these radiative processes depend on the collision (note these are plasma “collisions”) of an electron and an ion. Due to the very low density of the plasma all the ions excited by collisions have sufficient time for radiative de-excitation before a second de-exciting or further exciting collisions occur. Thus contrary to laboratory plasmas, where slow transitions are “forbidden” and the corresponding excited states are much more rapidly de-excited by electronic collision, all “forbidden” transitions actually happen in the ICM plasma.
This leads to a scenario where all exciting, recombining, and bremsstrahlung causing collisions lead to the radiation of a photon, which is referred to as the thin plasma radiation limit (or “coronal limit”, as similar conditions prevail in the solar corona).

These collision rates are in general a function of temperature (for thermal plasma) and the outcome is directly proportional to the electron density. The shape of the resulting spectrum is therefore a function of the temperature and chemical composition and its normalization is directly proportional to the electron density and the ion density.

So in equilibrium conditions we can assume that the number $X^i$ of ions $i$ of the species $X$ created or destroyed by ionization and by recombination coincide:

$$\left[ D(X^i) + C(X^{i-1}) \right] n(X^i) \hbar \omega = D(X^{i-1}) n(X^{i-1}) \hbar \omega + C(X^i) n(X^{i+1}) \hbar \omega$$

where $D(X)$: coefficient of destruction of the specie by collisional ionization

$C(X)$: coefficient of creation of the specie by recombination

where the left member corresponds to destruction of ions $X^i$ and the right hand member to the creation of ions $X^i$. The electron density obviously disappears from the equation, and the only relevant dependence is on the plasma temperature $T_g$ that is implicit in all coefficients $C(X)$ and $D(X)$. For any atomic species the fraction of atoms $X^i$ in the i-th ionization level is maximum at a given temperature value about equal to the ionization potential. This complex set of functions is reported in Fig. 4.

Figure 4. Line equivalent width as a function of plasma temperature in a condition of collisional equilibrium. As we see, the emission line energy depends on the ionization status of the atom, for example the 6.8 keV Iron K line ranges from 6.7 to 6.9 keV at increasing ionization of the Iron.
For example, at a plasma temperature of $T_g = 10^8 \text{ K}$ (or $kT_g = 10 \text{ keV}$), the Iron is partly totally ionized, partly hydrogenic (1 electron in the orbitals), partly Helium-like (2 electrons).

### 4.2.2 The X-ray emission composite

In X-ray astronomy, the following rules are useful to remember:

$$
912 \text{ A} = 13.6 \text{ eV} \Rightarrow 1 \text{ eV} = 10000 \text{ A} \Rightarrow 1 \text{ keV} = 10 \text{ A}
$$

$$
kT = 1.3810^{-16} T(K) \Rightarrow 1 \text{ keV} = 10^7 \text{ K} \Rightarrow 10 \text{ keV} = 10^8 \text{ K}
$$

Assuming the above ionization equilibrium, the next step is to calculate the spectral emission. Given the characteristic plasma temperatures (see Sect. 3), most of the line and continuum emissions will fall in X-rays. As we already saw, we have 3 contributions to radiation:

- free-free continuum emission following rules discussed in Sect. 2 (**continuum**)
- free-bound radiative recombination (**edges**)
- bound-bound transitions producing emission lines (**lines**).

For all these processes the emissivity is proportional to the ionic number density $n(X^i)$, the electronic number density $n_e$, and depends on temperature:

$$
\varepsilon_e = \sum_{X,i} \Lambda(X^i, T_g) \cdot n(X^i)n_e
$$

where $\Lambda(X^i, T_g)$ is the emissivity per ion and electron.

The results about the total emitted spectrum are effectively summarized in a simulation in Fig. 5. Fig. 6 shows some observational results of high-resolution spectroscopy on real clusters.
Figure 5. Predicted X-ray spectrum by a plasma at various temperatures. Both the contributions of continuum, emission lines and edges are shown. The model assumes an isothermal plasma inside a volume of radius of 0.5 Mpc and proton number density of $10^3$ particles per cubic centimeter. Only the strongest lines are reported. [From Sarazin & Bahcall].
4.2.3 The IC plasma metallicity

Among the nice features of the coronal line emission, an interesting aspect concerns the ease of measuring abundances from EW line measurements in a direct way. Indeed, as far as the line emission is concerned, under the hypothesis of collisional equilibrium, eq. (14) can be re-written as

$$\varepsilon_v = n_p n_e \sum_{X,i} \left( \frac{n(X^i)}{n_H} \right) \Lambda(X^i, T_g)$$
where the quantity \( n(X^i)/n_{H} \) is the abundance of the i-th ionization specie of the element with respect to hydrogen and the emissivity \( \Lambda(X^i, T_g) \) depends purely on the plasma temperature. Note that eq. (15) contains the same proportionality to \( n_{\rho} n_{e} \) that is also contained in the free-free emissivity, eq. (2.8). Therefore if we take the ratio of line intensity to continuum intensity for a given line of a given specie, this offers a direct measure of the ion abundance, once the plasma temperature is known. The latter is easily measured from the exponential cutoff of the continuum emission for example. Once integrated over the line profile, this ratio is nothing but the line Equivalent Width (EW):

\[
EW = \int \left[ \frac{I_v - I_0}{I_v} \right] d(h\nu)
\]

In conclusion, a simple measurement as that of the line EW for a line of a given ion allows us a direct estimate of the abundance of that ion compared to hydrogen. Summing over all ionization species of an element provides us with that total element abundance.

Observational results concerning metal abundances in local low-redshift clusters, in particular about the easiest-to-measure Fe abundance, are reported in Fig. 7.

On average, metal abundances in local clusters concentrate around the value of 0.3-0.4 the solar abundance.

![Figure 7. Observed Iron abundances in local clusters. Average values are about 0.4 the solar value.](image)
4.2.4 Metallicity properties and evolution

Cluster data obtained by ASCA were consistent with no evolution in the ICM metallicity out to redshift $z \sim 0.4$. More recent observations with Chandra, however, indicate a significant evolution (Balestra et al. 2007; Maughan et al. 2008). Balestra et al. (2007), who investigated the Fe abundance evolution on a sample of 56 clusters, found that while in the redshift range of $z = 0.3–0.5$ the average ICM Fe abundance is $\approx 0.4$ Solar, above redshift $z \sim 0.5$ the metallicity drops to $\approx 0.25$ Solar. Maughan et al. (2008) looked at a larger sample of 116 clusters at $0.1 < z < 1.3$ in the Chandra archive and essentially confirmed the results of Balestra et al. (2007). They found that the abundances drop by $\sim 50\%$ between $z \sim 0.1$ and $z \sim 1$ and the evolution is still present if the cluster cores (the inner $0.15 R_{500}$) are excluded from the analysis. This result indicates that the abundance drop is not due to the lack of strong Cooling Flows at large redshifts (see Sect. 4.3.2).

Balestra et al. (2007) also found a trend of the Fe abundance with the cluster temperature. Within $(0.15–0.3)R_{\text{vir}}$ in clusters with $T$ below 5 keV the Fe abundance is on average a factor of $\sim 2$ larger than in the hotter clusters. The Fe abundance values measured within $0.2R_{500}$ for the sample of 22 low redshift clusters analyzed by de Plaa et al. (2007) show the same trend with the cluster temperature: while in hot massive clusters ($kT \approx 5$ keV) the Fe abundance seems to be constant and equal to $\sim 0.3$ solar, for cooler clusters, in the temperature range of 2–4 keV, the Fe abundance shows a range of values between 0.2 and 0.9 solar. This trend is probably linked to the changing stellar mass over gas mass ratio in clusters and to the history of star-formation in cluster galaxies.

4.3 Distribution and structure of the IC plasma

4.3.1 Direct observational approach

The X-ray image of the Coma Cluster in Figure 8 demonstrates the power of X-ray astronomy in the study of clusters of galaxies. Intense X-ray emission is a common feature of rich clusters of galaxies, the emission being the bremsstrahlung of hot intra-cluster gas, as inferred from the extended nature of the emission and from the detection of the highly ionised iron line Fe xxvi in their X-ray spectra (Mitchell et al., 1976). These X-ray observations provide a very powerful probe of the gravitational potential of the cluster enabling the distribution of both the hot gas and the total gravitating mass to be determined. The cluster is assumed to be spherically symmetric and the gas in hydrostatic equilibrium within the gravitational potential defined by the total mass distribution in the cluster, that is, by the sum of the visible and dark matter
as well as the intra-cluster gaseous mass. If $P$ is the pressure of the gas and $\rho$ its density, both of which vary with position within the cluster, the requirement of hydrostatic equilibrium from eq. 3.4 Sect.3, with $v=0$, is:

$$\vec{\nabla}P = \vec{f} = \rho \vec{g}, \quad \Rightarrow \quad \frac{1}{\rho} \frac{dP}{dr} = -\nabla \phi(r) = -\frac{GM(<r)}{r^2}$$

The pressure is related to the local gas density and temperature $T$ by the perfect gas law $P = \rho Tk/\mu m_H$, where $m_H$ is the mass of the hydrogen atom and $\mu$ is the mean molecular weight of the gas. For a fully ionised gas with the standard cosmic abundance of the elements, a suitable value is $\mu = 0.6$. Differentiating the perfect gas law with respect to $r$ and substituting into (25) gives

$$\frac{kT}{\mu m_H} \left( \frac{1}{\rho} \frac{d\rho}{dr} + \frac{1}{T} \frac{dT}{dr} \right) = -\frac{GM(<r)}{r^2}$$

So, we can achieve from this a first guess about the integrated mass distribution:

$$M(<r) = \frac{kT}{G \mu m_H} \left( \frac{d\ln \rho}{d\ln r} + \frac{d\ln T}{d\ln r} \right).$$

Therefore, in its most simplified and concise form this equation for the total gravitating mass within the radius $R$ can be re-written, assuming the usual King profile, as:

$$M_{grav} \approx 10^{15} M_\odot \cdot \beta_{fit} \cdot T_{10keV} \cdot R_{Mpc}$$

with obvious meaning of the symbols and where $\beta_{fit}$ is a parameter of order unity to be discussed in Sect. 4.3.3. The same relation is also used to estimate the total gas mass contained in massive elliptical galaxies, that also contain hot plasma.

The overall integrated mass distribution within the cluster can be determined if the variations of the gas density and temperature with radius are known. Assuming that the cluster is spherically symmetric, these can be derived, in principle, from high sensitivity X-ray intensity and spectral observations. How can we obtain this information?

Consider that a suitable form for the bremsstrahlung spectral emissivity of a plasma, which we already obtained in (2.7-2.13), as a function of the particle density $n = \rho/\mu m_H$ and temperature $T$, is:

$$j_{bf} = \frac{dW}{dV dt dv} = 6.810^{-38} Z^2 n_e n_i T^{-1/2} \exp(-\hbar \nu/kT) \varpi_{bf} \text{ [erg / cm}^3 / \text{s / Hz]}$$
Now, high quality X-ray observations of the plasma emission, in principle, allow us to obtain $j_{ff}$ hence the plasma density and temperature. This would require on one side precise X-ray spectral measurements to determine the temperature distribution of the gas from the location of the spectral cut-off. Once this is done, the column density of the hot gas can be obtained in principle from the X-ray surface brightness measurements.

Unfortunately, such X-ray observations offer a highly integrated information on the distribution of the plasma quantities. Indeed the spectral emissivity has to be integrated along the line of sight through the cluster as from the geometry illustrated in the graph below. Performing this integration along the line-of-sight we get the X-ray luminosity within the coronal shell

$$I_{\nu}(b) = \frac{I_{\nu}(b)}{2\pi b \cdot db} = \frac{1}{2\pi b \cdot db} \int dV \cdot j_{ff}(r)$$

considering that the integral along the $x = \sqrt{r^2 - b^2}$ coordinate can be put as $dx = 2rdr/\sqrt{r^2 - b^2}$, so that $dV = 2\pi b \cdot db \cdot dx$. 

Figure 8. An X-ray image of the Coma Cluster of galaxies with the XMM-Newton Observatory (Longair HEA).
Figure 10. Determination of the physical properties of the cluster A1413 from high quality X-ray imaging and spectroscopy by the XMM-Newton X-ray Observatory. (a) The X-ray brightness distribution as a function of distance from the centre of the cluster. (b) The projected radial distribution of the temperature of the gas. (c) The integrated mass distribution as a function of distance from the centre. (d) The fraction of gas density to total mass density $f_{\text{gas}}$ within the cluster as a function of overdensity $\delta$ relative to the critical cosmological density (Pratt and Arnaud, 2002). The best fitting models are based on a hydrostatic isothermal scheme (Sect. 4.4.3).
and converting it into an intensity, the observed surface brightness at projected radius $b$ from the cluster center is

$$I_v(b) = \frac{1}{2\pi} \int_b^\infty \frac{j_\| (r)r}{(r^2 - b^2)^{1/2}} \, dr,$$  \hspace{1cm} 29

Eq. (29) is an Abel integral that has a formal inversion in the solution

$$j_\| (r) = \frac{4}{r} \frac{d}{dr} \int_b^\infty \frac{I_v(b)b}{(b^2 - r^2)^{1/2}} \, db.$$ \hspace{1cm} 30

(Cavaliere 1980).

This path has been pursued in a very few special cases in the presence of exceptionally deep and detailed observations (see e.g. Fig. 10). Unfortunately, not always mathematics can help us! Indeed, the procedure of numerical inversion entails very high statistical uncertainties, inherent in particular to the radial derivation of eq. 30, unless the statistical errors of the observations are extremely small.

For these reasons, in the vast majority of cases a more realistic approach is to refer to models of the plasma distribution, as discussed below.

### 4.3.2 Models of the plasma’s spatial distribution. Dynamical models

Historically, various models have been considered to represent the IC plasma distribution. We review the most significant here.

1. **Hydrostatic polytropic models.** For these kind of models the relationship between $P$ and $T$ in eq. (25) is generalized to:

$$\frac{P}{P_0} = \left( \frac{\rho}{\rho_0} \right)^\gamma,$$

where $\gamma$ is the polytropic index, expressing the dynamical status of the plasma. A condition of isothermality, $\gamma = 1$, requires an efficient heat conduction across the cluster. Adiabaticity, $\gamma = 5/3$, implies that heat conduction and radiative processes are very inefficient. A special important case of such polytropic models is the hydrostatic isothermal, with $\gamma = 1$, that is discussed in detail in Sect. 4.3.3 below.
2. **Hydrostatic self-gravitating models.** In this case the gravitational field is dominated by the plasma itself, such that

\[
\frac{1}{\rho} \frac{dP}{dr} = -\frac{4\pi G}{r^2} \int \rho_{\text{gas}} r^2 dr
\]

a very simple model with just 3 parameters, \( \rho_0, T_0, R_C \).

3. **Non-hydrostatic infall models.** The plasma here is assumed not to be in hydrostatic equilibrium inside the gravitational potential, but in an infall condition caused by primordial gas falling into the cluster. A critical parameter here is the mass infall rate \( \dot{M} \). The resulting structure would be adiabatic, \( \gamma = 5/3 \).

4. **Non-hydrostatic wind models.** The structure here is dominated by a global outflow of gas due to (enormous) energy injection by some sources inside the cluster (as discussed in Sect. 4.4). The resulting structure would be isothermal, but with an enormous energy request (\( 10^{46} \text{ erg} / \text{s} \)) and large mass outflow rates (\( \dot{M} \approx 10^{3-4} M_\odot / \text{yr} \)).

5. **Cooling-flow models.** If the density of the hot intra-cluster gas is sufficiently high, the gas may cool over cosmological time-scales. At high enough temperatures, the principal radiation loss mechanism is free-free emission, and line emission at lower temperatures. From eqs. (2.10 and 2.34) the cooling time is \( t_{ff} \approx 10^{12} \text{ yrs} \left( T/10^8 \right)^{1/2} \left( 10^{-4} / n_e \right)^{-1} \). For lower temperatures and/or higher densities the cooling time may become shorter than the Hubble time, that implies an hydrostatics unbalance in the plasma, that may frequently happens in the cluster central regions (where the density is the highest). This brings a continuous flow of gas in the central region, with a roughly isothermal structure again because the adiabatic compression is counter-balanced by radiative outflow. Observations indicate this structure for the cluster IC plasma for many clusters of galaxies. We define as cooling radius the distance \( r_{cool} \) from the center at which the free-free cooling timescale is equal to the Hubble time, that is the \( r \) value for which \( t_{ff} \approx 10^{10} \text{ yrs} \):

\[
\left[ n_e (r_{cool}) / 10^{-2.5} \right] \approx \left[ T(r_{cool}) / 10^7 \right]^{1/2}
\]
In regions outside $r_{\text{cool}}$, the plasma is in equilibrium for an almost infinite time, while it cools and flows inside. This event is illustrated in Figure 11 for the cluster Abell 478. The ROSAT observations were de-projected to determine mean values of the density and temperature of the gas as a function of radial distance from the center. Figure 11 shows that the temperature decreases towards the central regions, while the electron density increases to values greater than $10^{-4}$ cm$^{-3}$ at the very center. At a radius of 200 kpc, the electron temperature is $T = 7 \times 10^7$ K and the electron density $N_e = 8 \times 10^{-3}$ cm$^{-3}$. Inserting these values into (4.33) corresponds to a cooling time of $10^{10}$ years. Outside this radius, the temperature of the gas is constant. As a result, matter drifts slowly in through the surface at radius $r_{\text{cool}} \approx 200$ kpc, at which the cooling time of the gas is equal to the age of the cluster. The X-ray luminosity of the cooling flow results from the internal energy of each element of the gas as well as the work done as it drifts slowly in towards the central regions whilst maintaining hydrostatic equilibrium. For this cluster the central cooling flow
results in a large mass inflow rate of about 600–800 \( M_\odot / \text{yr} \), while more typical values for many cooling clusters are around 100–300 \( M_\odot / \text{yr} \).

### 4.3.3 Models of the plasma’s spatial distribution. Hydrostatic isothermal models

As we have seen in Sect. 4.1.2, the elastic collision timescales in the plasma are much shorter than those of heating or cooling or other dynamical processes. For this fluid, we can immediately estimate the time required for the sound waves to cross the cluster. From eq. (3.39)

\[
t_s = \frac{R_c}{c_s} \approx 6.6 \times 10^8 \text{ yrs} \left[ \frac{T}{10^8 \text{ K}} \right]^{-1/2} \frac{R_c}{1 \text{Mpc}}
\]

a time that is much shorter than the probable age of the cluster. Then the gas may be assumed to be in hydrostatic equilibrium. If the contribution of the IC gas to the total gravitating mass of the cluster is negligible, then the gas distribution is only determined by the cluster potential \( \phi(r) \) and by the temperature distribution.

If, for the above-mentioned reasons, at least a rough iso-thermality condition applies (\( P \propto \rho, \gamma = 1 \)), then we have from eq. (25) a first condition on the plasma distribution as:

\[
\frac{1}{\rho} \frac{d\rho}{dr} = -\frac{\mu m_{\text{H}}}{kT} \frac{d\phi(r)}{dr}.
\]

At the same time, also the motions of galaxies feel the same gravitational potential, and their spatial distribution will depend on it and on \( \sigma_r \):

\[
\frac{1}{\rho_g} \frac{d\rho_g}{dr} = -\frac{1}{\sigma_r^2} \frac{d\phi(r)}{dr}
\]

where \( \rho_g(r) \) is the volume density of galaxies. This means that both galaxies and plasma particles feel the same gravitational potential. It is immediate to eliminate the gravitational potential from 36 and 37, to obtain

\[
\frac{d\ln \rho}{dr} = \frac{\mu m_{\text{H}} \sigma_r^2}{kT} \frac{d\ln \rho_g}{dr} = \beta \frac{d\ln \rho_g}{dr},
\]

where

\[
\beta \equiv \frac{\mu m_{\text{H}} \sigma_r^2}{kT},
\]

and whose solution reads
If instead the isothermality condition does not apply, then a more general solution to eq. (25) than eq. (37) should be considered. This is done by adopting the relation in eq. (31) in its more general form \( p \propto \rho^\gamma \), with \( \gamma \neq 1 \), and proceeding to the integration of (25) we obtain:

\[
\frac{\rho}{\rho_0} = \left( 1 + \beta \frac{\gamma - 1}{\gamma} \ln \left[ \frac{\rho_G}{\rho_{G,0}} \right] \right)^{\frac{1}{\gamma - 1}}, \quad \gamma \neq 1
\]

Our next step is then to consider as a tracer of the total gravitational potential the distribution of galaxies \( \rho_G(r) \). At least for dynamically relaxed clusters, it is a likely good assumption that the cluster potential is that of a self-gravitating isothermal sphere, the well-known King model. A good approximation to the latter, able to reproduce fairly well the matter distribution in the cluster, is the analytic approximation to the King profile:

\[
\rho_G(r) = \rho_{G,0} \left[ 1 + x^2 \right]^{-3/2}, \quad \text{with} \quad x = \frac{r}{r_c}, \quad \text{whose integral gives}
\]

\[
M(<r) = 4\pi \rho_0 r_c^3 \left\{ \ln \left[ x + \left( 1 + x^2 \right)^{1/2} \right] - x \left( 1 + x^2 \right)^{-1/2} \right\}
\]

\( r_c \approx 250 \text{ kpc} \) being the cluster core and \( \rho_0 \) the central mass density. With reference to galaxies, these two quantities are simply connected to the line-of-sight galaxy velocity dispersion

\[
\sigma_r^2 = \frac{4\pi G \rho_0 r_c^2}{9} = -\frac{\phi_0}{9}.
\]

For the isothermal model, this together with eq. (37) implies, as a simple model solution to the plasma density distribution:

\[
\rho(r) = \rho_0 \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{-3\beta/2}
\]

with
\[ \beta = \frac{\mu m_H \sigma_r^2}{kT} = 0.76 \left( \frac{\sigma_r}{10^3 \text{ km/s}} \right)^2 \left( \frac{T}{10^8 \text{ K}} \right)^{-1} \]

After insertion into (29), we obtain a model representation for the surface brightness distribution as:

\[ I_X(b) = I_{X0} \left[ 1 + \left( \frac{b}{r_C} \right)^2 \right]^{-3\beta+1/2} \]

where \( b \) is the impact parameter and \( I_{X0} \) is the brightness at the cluster center.

Interesting insights can be obtained by comparison of the parameter inferred from eq. (42), \( \beta_{\text{fit}} \), and that from (41), \( \beta_{\text{spec}} \) (it was sometimes found that the two were not consistent, but it was later realized that it was due to a mis-representation of the total matter profile).

This very simple model appears to give a fairly accurate description of the observational data on the X-ray surface brightness distribution in rich regular clusters. These observations are then consistent with values of the \( \beta \) parameter around unity, which implies also a condition fairly close to iso-thermality in the plasma gas,

\[ \gamma \approx 1 - 1.2, \quad \beta \approx 1. \]

Some results of the application of this model to cluster observations are illustrated in Figure 10 above.

### 4.3.4 Other models: conclusions

To conclude, hydrostatic, mostly isothermal, models offer a good representation of the plasma distribution. Concerning the other dynamical models in Sect. 4.3.2:

2. Hydrostatic self-gravitating models are completely inconsistent with the observations, as they require 10-20 times more plasma mass than would be consistent with the X-ray observations (see Sect. 4.3.4 below).

3. Non-hydrostatic infall models imply a roughly adiabatic structure of the plasma, once again inconsistent with the indications of eq. 43. Among other things, this implies that Dark Matter in clusters of galaxies is not in the form of a hot plasma.

4. Non-hydrostatic wind models, while consistent with the isothermality evidence, require instead an excessive and implausible energetics (to expel the gas).
5. Cooling flow models. As anticipated, more or less direct evidence in favor of cooling flows are reported for many X-ray clusters of galaxies. These models are perfectly consistent with the successful hydrostatic isothermal model over the bulk of the cluster volume outside the radius corresponding to $r_{cool}$ (eq. 33), while an hydrostatic unbalance producing a cooling flow happens only in the very inner regions. While this model appears to be successful in principle, it presents some complications in the fact that it makes some predictions about a strong line emission flux by high-ionization lines in the cooling flow region that are not met by the observations. Line emission would be expected in the presence of a low-temperature plasma, like discussed in Sect. 4.2.2 and Fig. 6, that is at about $T \approx 10^7 \, K$. The fact that such strong emission lines are either not observed or faint implies that the plasma never cools down to that temperature limit. The Chandra X-ray Observatory has shown that the cooling gas in the central regions of a number of clusters is perturbed by the presence of radio lobes associated with recent radio source events. In a very long X-ray exposure with the Chandra observatory, Fabian and his colleagues identified what they interpret as isothermal sound waves produced by the weak shock waves associated with the expanding radio lobes. They showed that the energy injected into the IC gas by these sound waves can balance the radiative cooling of the flow (Fabian et al., 2006).

In conclusion, the gas mass accumulated during a Hubble time would be of the order of $10^{12}$ to a few times $10^{12} \, M_\odot$, corresponding to a whole to several massive galaxies. Part of this material is likely collapsing onto a nuclear AGN and activating it, with the result of stopping further accretion and accumulation.
because of the energy (both mechanical and radiative) injection by the active nucleus. This is a classical feedback process that self-regulates the cooling flow. All this has important consequences for cosmology and galaxy formation.

4.3.4 Mass distributions of the galaxies, plasma and Dark Matter distributions in clusters of galaxies

Let us now go back to our equation (27). What is remarkable about the X-ray observations of galaxy clusters based on the free-free emission of their hot plasma content is that high quality observations provide simultaneous data (photometric imaging and spectroscopy) useful for determining, on one side, the plasma content of the cluster based on the reconstructed free-free emissivity. On the other side, they also constrain the total (normal and dark) matter content of the cluster and its distribution.

An example of the mass reconstructions of the various components for the Perseus cluster of galaxies as a function of the radial coordinate based on the ROSAT X-ray observatory data (Boeringer 1995) is reported in the figure below. The mass in the visible part of galaxies, in gaseous mass and total gravitating mass (with uncertainty bands) is shown.

In summary, X-ray observations add valuable information about the mass distributions of the various components to other probes (gravitational lensing, galaxy distributions). All these data indicate the following general trends.

- The galaxies follow the typical King profile $\rho_G \propto r^{-3}$ and make the steepest centrally concentrated component.

- Dark matter, according to various probes, is intermediate, $\rho_{DM} \propto r^{-2}$. More specifically, an important cosmological result of X-ray cluster observations is that the relaxed regular clusters appear to follow closely the Dark Matter halo mass distribution predicted by Navarro, Frank and White (1995, the NFW profile distribution) based on numerical simulations of the Cold Dark Matter collapse. An illustration of such good fits is reported in Fig. 13.

- X-ray plasma: for relaxed systems (like rich clusters), $\rho_{plasma} \propto r^{-2}$; for irregular clusters $\rho_{plasma} \propto r^{-1}$. Note that irregular clusters tend to be of lower mass, regular relaxed clusters are typically massive. The shallower X-ray profile of the irregulars has likely to do with energy injection onto the plasma, see Sect. 4.5
Figure 12. Distribution of masses of the various components in the Perseus cluster of galaxies as a function of the radial coordinate. Dark matter can be obtained by subtraction from the total of other components.

Figure 13. Fits to the Dark Matter mass distribution based on the NFW model compared with cluster observations by the XMM-Newton observatory.

All this has fundamental implications for our understanding of structure formation.

The other fundamental implication is that, because the gas total mass is 5-7 times larger than the mass in galaxies, the total content (in mass) of heavy elements is factors 2-3 more in the ICM than it is contained in stars and galaxies. The bulk of the elements produced by stars is lost in favor of the diffuse medium.
4.4 The local baryon budget

All data and analyses that we have discussed in this Section, together with independent estimates, allow us to draw interesting conclusions about where are the baryons at the present cosmic time (see Driver et al. 2017 MNRAS).

Figure 20. Scheme showing the percentages of where the baryonic matter are found to reside in the different cosmic environments. The percentages refer to the global baryonic matter content of the Universe, which in turn amounts to 4.5% of the closure density according to the cosmological studies. The various acronyms and relevant papers are as follows.
UNBOUND (diffuse):

- **Hot Plasma** (28 per cent; Fukugita, Hogan & Peebles 1998; Shull, Smith & Danforth 2012). Hot plasmas present inside the cosmological structures (galaxy clusters, groups, galaxies themselves. This is a component easily detectable via X-ray observations.

- **Warm Hot Intergalactic Medium** (29 per cent; Shull, Smith & Danforth 2012). This is a component not detectable via the study of its X-ray emission because it is too faint. This component can be detected in absorption looking at the X-ray absorption lines in deep very high-quality spectra of background luminous AGNs.

BOUND TO CLUSTER AND GROUP HALOSs:

- The **intra-cluster light** (4 per cent; Shull, Smith & Danforth 2012)
- The intra-group light (< 1 per cent; Driver et al. 2016)

BOUND TO GALAXY HALOS:

- **Stars** (7 per cent, Baldry, Glazebrook & Driver 2008, 2012; Peng et al. 2010; Moett al. 2016; Wright et al. 2017a)
- **Neutral gas** (2 per cent; Zwaan et al. 2005; Martin et al. 2010; Delhaize et al. 2013; Martindale et al. 2017)
- **Circum-galactic medium** (5 per cent Shull, Smith & Danforth 2012; Stocke et al. 2013)
- **Molecular gas** (0.2 per cent; Keres et al., 2003; Walter et al. 2014)
- dust (0.1 per cent Vlahakis, Dunne & Eales 2005; Driver et al. 2007; Dunne et al. 2011; Clemens et al. 2013; Beeston et al. 2017)
- **Super-massive Black Holes** (**SMBHs**) (0.01 per cent Shankar et al. 2004; Graham et al. 2007; Vika et al. 2009; Mutlu Pakdil, Seigar & Davis 2016)

UNACCOUNTED FOR:

- **Missing baryons** (25 per cent; see also Shull, Smith & Danforth 2012). This is the amount of baryonic matter needed to explain the total baryon content of the Universe based on cosmological observations.
4.5 Thermodynamics of the IC plasma

4.5.1 Plasma Cooling

For plasma temperatures $T_g > 10^7 \, K$, the most important cooling process is due to free-free emission.

For $T_g < 10^7 \, K$, the line emission is responsible for most of the cooling.

The differences in the efficiency of line-cooling and continuum-cooling are clearly revealed by the graph in Fig. 6.

We see that line cooling is extremely more efficient in cooling gas than the cooling by the continuum emission. Indeed, if the plasma temperature approaches $10^8 \, K$, there is no more chance for the plasma to cool significantly over a Hubble time, as discussed in Sect. 2.7.3.

Figure 14. Cooling rate of hot plasma as a function of the plasma temperature. The contribution to the cooling by the ions of different important abundant elements is indicated (Böhringer and Hensler 1989). Most of this contribution is in the form of line radiation, which is by far the dominant form of radiation in the temperature range from about $10^4$ to $2 \times 10^6 \, K$. At higher temperatures over most of the regime of interest for the ICM the Bremsstrahlung contribution dominates.
4.5.2 Plasma Heating

Today there is no evidence for processes currently active in producing a heating of the plasma to the observed temperatures. However, many such processes are likely to have been active during the past history of clusters. The plasma itself is a magnificent record of all these processes: because the IC plasma is very hot and low-density, the plasma we observe today contains the integral of all those processes.

**Infall of primordial gas and heating by adiabatic compression.**

If the cluster gravitational potential was formed and stabilized before the infall of external (presumably primordial) gas, then the infall will be essentially adiabatic because the cooling time is long. This may have heated the plasma to a temperature that can then be calculated from the virial condition:

\[
\frac{3}{2} \frac{kT}{\mu m_H} \approx -\phi_0
\]

where for an isothermal sphere \( \phi_0 = -9\sigma_r^2 \). For such an adiabatic collapse we can then get a temperature of

\[
T_g \approx 5 \times 10^8 \left( \frac{\sigma_r}{1000 \text{ km/s}} \right)^2 \text{ K}
\]

This is the maximum temperature achievable. It is more likely that the plasma has been infalling into the cluster when the gravitational potential \( \phi_0 \) was not yet completely formed, or has been released by the same galaxies belonging to the cluster (hence it has underwent a lower compression), or more likely it has been accumulated during the whole cluster collapse.

On the other hand, if the gas has experienced the same **violent relaxation**\(^2\) that ruled the dynamical status of the galaxies, the plasma temperature will correspond to the kinetic energy per unit mass of the galaxies:

\[
\frac{kT}{\mu m_{\text{H}}} \approx \sigma_r^2 \Rightarrow T_g \approx 10^8 \text{ K}
\]

quite more in agreement with the observations.

\(^2\) Violent relaxation is a statistical reshuffling of the kinetic energy per unit mass of galaxies in their orbital motions in the presence of violent fluctuations of the cluster gravitational potential likely occurring during the cluster formation. To a good approximation, this process was found to produce a redistribution of the (kinetic) energy per unit mass, \( kT/m \), or \( \sigma^2 \).
**Heating by galactic winds.**

The demonstrated presence of metals in the IC plasma indicates that at least a fraction of the gas should have been processed by the galaxies. The contributions to the final thermal energy are twofold:

a) the energy with which the gas is ejected by the galaxies as galactic wind

b) the energy due to the motion of galaxies with respect to the frame inertial with the cluster.

In general the mechanism to convert the ordered kinetic energy into thermal energy is related with the generation of shocks waves, that are originated by the interaction between the expelled gas and that already present inside the cluster.

Already the contribution of (b) could explain the high observed temperatures:

\[ kT_g = \mu m_H \sigma_r^2. \]  

In addition, also contribution (a) can be significant:

\[ kT_g = \mu m_H (200 \text{ km / s})^2 \]

where the velocity is that characteristic of the internal motions of stars in galaxies. In fact, much larger velocities can be found in galactic winds. Supernovae explosions (those responsible for the production of heavy elements) eject enriched gas with velocities of \( V_{SN} \approx 10^4 \text{ km / s} \), that colliding with the ambient gas produce strong and highly supersonic shocks (Mach number \( \mathcal{M} \gg 1 \)). The resulting temperature is immediately calculated from eq. (3.95)

\[ T = \frac{1}{5} \frac{\mu m_H}{k} V_{SN}^2 \approx 2 \times 10^9 \text{ K} \]

hence far more than sufficient, unless the dilution of the ejected material by mixing with the ambient medium is very high (which is obviously likely).

**Heating due to the galactic motions.**

It is very difficult to estimate this potential source of heating, because of various reasons.

1. Motions of galaxies are just typically transonic:

\[ c_s = 1500 \text{ km / s} \left( \frac{T_g}{10^8} \right)^{1/2} \]
so assuming a typical orbital velocity for galaxies the velocity dispersion in the
cluster of $\sigma_r \approx 1000\,\text{km}/\text{s}$, so that $\langle v^2 \rangle^{1/2} = 3 \sigma_r^{1/2} = 1700\,\text{km}/\text{s}$, we get

$$\langle \mathcal{M} \rangle = 1.5$$

such that this motion is neither highly super-sonic (a situation that would allow us to
treat it in the easy strong-shock limit $\mathcal{M} \gg 1$) nor sub-sonic (that would be suited to
the incompressible fluid treatment).

2. The mean free path of the particles is of the same size of the galaxies
($\lambda \approx 23\,\text{kpc}$), so it is unclear how to treat the interaction between the
galaxies and the plasma, if the gas behaves as a non-collisional fluid in his
interaction with galaxies, or as a normal fluid.

3. It is unclear the role of viscosity, so unclear if to treat the motions in the fluid
as laminar or turbulent. The shear viscosity (as we will discuss later in Sect.
10) can be expressed as

$$\vec{F}_{\text{visc}} \sim \eta \nabla^2 \vec{v}$$

$\eta$: dynamical viscosity. For an ionized plasma:

$$\eta = \frac{1}{3} m_i n_i \langle v_i \rangle \lambda_i$$

(as for the thermal conduction, the shear viscosity does not depend on the plasma
density, while it depends strongly on the temperature $T_g$). Here too the ions are the
main responsible.

To assess the importance of viscosity, hence the laminar or turbulent structure of
the fluid, we should refer to the Reynold number

$$R_e = \frac{\rho_g \ell_g / \eta}{3 \cdot \mathcal{M} \left( \frac{\ell_g}{\lambda_i} \right)}$$

$\ell_g$: size of the object in motion

$$\rho_g = m_i n_i$$

Now, if $R_e \gg 1 \Rightarrow$ turbulent motions.

In our case, however, we have $R_e \approx 5$, so the motions are likely laminar, but it is
not sure.

4. Finally, the nature of the drag (friction) forces, depends if galaxies are rich
of interstellar gas or not. In the second case the interaction is purely
gravitational. Let us calculate it, by considering the maximum amount of
energy that can be produced by the viscous forces:
The first factor has the units of a pressure, times an area gives a force, force times the velocity of the entity producing the force gives a power. Then this product quantifies the maximum power that can be extracted by the motion of galaxies, assuming that their cross-section is given by $\pi R_D^2$. Then (26) can be re-written as

$$\frac{dE}{dt} = \tilde{F}_{\text{visc}} \cdot \tilde{v} = \pi R_D^2 \cdot \rho_g v^3$$

$R_D$ is the effective radius for the interaction. In the case of a purely gravitational interaction, this quantity can be computed with reference to the theory of the dynamical friction produced by the motion of a massive object inside a distribution of lower mass particles. In such case, $R_D \ll \ell_g$. Otherwise, in the presence of an ISM inside the galaxy, or a galactic scale magnetic field, $R_D \approx \ell_g$.

Certainly in the former case these orbital motions cannot at all contribute to the plasma heating, $E/(dE/dt) \approx 10^{11} - 10^{12}$ yrs $>> t_H$. Instead, the friction has certainly a role to spoil galaxies of their ISM, but seem unlikely to heat the plasma. On this regard, there is a simple argument. The mass of the IC gas in clusters is about a factor 2 larger than the stellar mass in galaxies, so the thermal energy in the gas is larger than the kinetic energy in galaxies, and it is hard to imagine that galaxies might have lost most of such energy in favor of the gas (the velocity dispersion $\sigma_r$ should have been enormous in the past).

**Gas heating due to ultra-relativistic particle injection.**

Galaxy clusters often contain luminous radio-galaxies with steep radio indices (see Sect. 5) produced by ultra-relativistic electrons. These latter and the associated ions interact with the IC plasma and can heat it. Clearly the rate of heating is expected to be proportional to the radio synchrotron luminosity of the galaxies. By integrating the observed radio luminosities in the frequency interval that is observable from Earth, we can easily estimate that such electrons are able to produce a completely negligible heating rate. Even adding the ion contribution (this not directly observable, see our discussion in Sect. 5) we do not get to significant values. Because radio observations are impossible below the frequency of $\nu = 10^7$ Hz, and because synchrotron self-absorption prevents observing typically below 100 MHz, a strong flux of low-energy electrons and protons from radio-galaxies might remain a
possibility. This would make an enormous energy request, not verifiable hence implausible.

From a general point of view, models of this kind suffer by two problems. 1) The global energetics request is enormous ($10^{63} - 10^{64}$ erg), there would be the needed concourse of too many radio-galaxies ($10^{60} - 10^{62}$ erg each one maximum) that are not observed (just one to a few radio-galaxies per cluster). 2) Radio-galaxies typically occupy a small fraction of the cluster volume, so it would be difficult to understand how this process could happen without generating substantial inhomogeneities in the X-ray surface brightness distribution.

However, in any case emissions (both radiant and mechanical) by Active Galactic Nuclei may have significant influence in heating the plasma in the core region of the cluster and limit by these means the cooling flows.

4.6 **Scaling Relations and Origin of the IC Plasma**

4.6.1 **Scaling relations**

Simple scaling relations between physical parameters of galaxy clusters would be expected if we assume that the cluster build-up and the plasma thermal structure are merely determined by gravitational processes (no other energy generation terms).

Galaxy clusters form from the gravitational collapse of overdense regions in the dark matter dominated mass density distribution in the Universe. The complex theories and studies of how the mass density field originates and evolves with cosmic time are discussed in the framework of the Theoretical Astrophysics and Cosmology course of the Master Degree. We just assume what is discussed there and consider here the implications of a simple adiabatic collapse of primordial plasma inside the DM halos of the forming clusters.

To summarize, an over-density of matter detaches from the Hubble expansion flow at some given epoch depending on the local density excess $\Delta = d \rho / \rho$. At some later stage, the matter inside this volume stops expanding and starts contracting. At about the stage of first collapse, the strongly varying and fluctuating gravitational potential generates a *violent relaxation* of the whole matter content, with an approximately virialized structure produced at the end.
The whole cluster then approaches an equilibrium configuration characterized by a virial relation (roughly corresponding to a re-distribution of kinetic energies per unit mass):

\[
\frac{E_{\text{kinetic}}}{\text{unit mass}} \approx v^2 \propto \sigma_{DM}^2 \quad \text{and} \quad \frac{E_{\text{kinetic}}}{\text{unit mass}} = -2\frac{E_{\text{potential}}}{\text{unit mass}} \propto \frac{GM}{R}
\]

where \( M \) is the total mass of the galaxy cluster including the DM. Similarly to the virial equilibrium of galaxies and DM particles, the ICM plasma thermalizes and attains a “virial temperature” which reflects the depth of the gravitational potential of the cluster. In the collapse process, the potential energy of the ICM is converted to internal heat (Sect. 4.4). If the gravitational potentials of clusters of different mass have a self-similar shape, as implied by numerical simulations of gravitational collapse (e.g. Navarro et al. 1995), then one finds the following self-similar relation between cluster mass and ICM temperature:

\[
T \propto \sigma_{DM}^2 \propto M/R \propto M^{2/3}
\]

where \( \sigma_{DM} \) is the velocity dispersion of the dark matter particles. (The latter relation is due to \( M \propto R^3 \) and \( \rho_{DM} = \text{const.} \) in the self-similar model).

Figure 15. Left The mass–temperature relation of a sample of regular clusters from (Arnaud et al. 2007). The best fitting slope (solid line) is 1.71 ± 0.09 slightly steeper than the self-similar relation. Right The mass–temperature relation of galaxy clusters and its evolution with redshift derived from two cluster samples at \( z \sim 0.05 \) and \( z \sim 0.55 \) (Kotov and Vikhlinin 2006). The relations have been displaced by a factor of 10 for better visibility. The model predicted redshift evolution (clusters of equal mass get hotter with look-back time) has been accounted for by scaling with the factor \( h(z) \). This predicted evolutionary trend is well supported by the data [Böhringer · Werner 2012].

4.34
In a similar fashion, two other relations are found, still under the assumption of a thermo-dynamics dominated by gravitation $^3$:

$$I_X \propto M^{4/3}$$  
$$I_X \propto T^2$$

How much do the observational data agree with such predictions? Indeed, to first order these model predictions are in fair agreement with observations. This is illustrated in Fig. 15, where the mass versus temperature relation is reported. As we see, the data show indeed an excellent correlation between the total cluster mass and the X-ray temperature of the plasma. The slope of the relation is not too much dissimilar to that in eq. 62, although it is a bit steeper (slope of about 1.7 against the 1.5 expected value). Some level of disagreement exists, but it is not dramatic.

The most serious observed deviation of data compared to the gravitationally-dominated thermodynamics is instead about the X-ray luminosity versus plasma temperature relation, as shown in Fig. 16. The observed relation is much steeper than expected, particularly for the lowest mass-luminosity systems (groups of galaxies).

$$^3$$ The argument runs as follows. The free-free emission implies that $I_X \propto V \rho_g^2 T^{1/2}$. But $T \propto \sigma_{\text{DR}}^2 \propto M/R \propto M^{2/3}$, $\rho_g \propto \rho_{\text{DM}} = \text{const}$, and $M \propto R^3 \propto V$. In conclusion, $I_X \propto V \rho_g^2 T^{1/2} \propto R^2 T^{1/2} \propto M \cdot M^{4/3} \propto M^{4/3} \propto T^2$. 

Figure 16. The (bolometric) luminosity–temperature relation for nearby and distant clusters and groups compiled from several sources (see Borgani et al. 2001b, Holden et al. 2002). The two dashed lines at $T > 2$ keV indicate the slope $\alpha = 3$, with normalization corresponding to the local $L_X - T$ relation (lower line) and to the relation of computed at $z = 1$. The dashed line at $T < 1$ keV shows the best–fitting relation found for groups by Helsdon & Ponman (2000).
The interpretation of these findings is that the thermo-dynamical structure of the plasma is not simply related to gravitational energy converted into heat, but additional heating sources are required. All the available data (including analyses considering the entropy associated to the plasma) suggest that there should have been a release of thermal energy to the plasma by some heating sources. These events, probably occurring in the distant past, may be due to massive stellar activity or AGN emission. The energetics requirement corresponds to about 1 keV of heat produced per nucleon.

The low-temperature systems (galaxy groups) are observed to have shallower central gas-density profiles than the hotter systems (regular clusters), which turns into an excess of entropy, particularly in low–luminosity and low-mass structures. This heating would both increase the plasma temperature $T_X$ and decrease $L_X$: it would increase the entropy of the ICM, preventing it from reaching a high central density during the cluster gravitational collapse, therefore decreasing the X-ray luminosity. For a fixed amount of extra energy per gas particle, this effect is stronger for poorer clusters, i.e. for objects whose virial temperature is comparable with the extra–heating temperature. As a result, the self–similar, gravitationally-dominated behavior of the ICM is expected to be better preserved in hotter, more massive systems, whereas it is broken for colder systems.

These considerations offer a good explanation of the disagreement with respect to the above mentioned expectations by the gravitationally-dominated thermodynamics. The mass-temperature relation shows a moderate disagreement in Fig. 15 because only the plasma temperature is involved. Instead the luminosity-temperature relation is seriously mismatched because two effects operate there in multiplication, one is about $T_X$ increase, the other is about a strong decrease of $L_X$ at low values of $T_X$ and cluster mass.

4.6.2 Origin and evolution of the IC plasma

Altogether, the IC plasma in clusters and groups is a fundamental astrophysical and cosmological component. It includes about 5-7 times the total mass of baryons condensed in stars. In addition, even if the metal content of this matter is about 0.3-0.4 the solar metallicity, the total content of heavy elements in this plasma is from 2 to 3 times the corresponding amount in stars.

It is then of great importance and interest to ascertain the origin and the history of this component, when it formed and when and how it has been enriched of the heavy elements.
Figure 17: The cluster cumulative number counts as a function of X-ray flux (log\(N - \log S\)) measured from different surveys.

Figure 18: Determinations of the local X-ray Luminosity Function of clusters from different samples (note this is scaled to \(H_0 = 50\) km s\(^{-1}\) Mpc\(^{-1}\)). The shape of this function is consistent with that expected by the Press & Schechter theory discussed in the Cosmology master course. [From Rosati et al 2002].
The simplest and most direct way to test the cosmological evolution of this plasma is to consider the number counts of the X-ray sources associated with galaxy clusters. This statistical test is particularly effective in consideration of the relative ease of detection of these X-ray sources, that are well concentrated and easily identifiable on top of the map’s background.

The number count test is shown in Fig. 17. To interpret it, we need some knowledge about the (local) luminosity functions of X-ray emitting clusters of galaxies, which is reported in Fig. 18. The latter appears to be fairly in keeping with expectations based on the Press-Schechter theory (Master Degree Cosmology course), note in particular the rather steep shape of the luminosity functions at low luminosities, \( \frac{dN}{dL_X} \propto L_X^2 \).

The number counts in Fig. 17, as well as the redshift-dependent luminosity functions shown in Fig. 19, indicate that up to \( z=1 \), there is no much evidence for evolution in the cluster population. Preliminary results indicate instead a decrease in the comoving volume density of the highest mass-luminosity clusters only at \( z>1 \).

These results are not easily reconciled with the predictions of $\Lambda CDM$ models, that would naturally expect cluster to form at moderate to low redshifts ($z<0.5\sim 1$).

As for the evolution of the plasma metallicity, we have discussed it in Sect. 4.2.4: there is a clear indication for a decrease of the metal content above \( z=0.5 \).

![Figure 19. X-ray Luminosity Function of distant clusters out to $z \approx 0.8$ compiled from various sources and compared with local XLFs. Numbers in parenthesis give the median redshift and number of clusters in each redshift bin.](image)
In conclusion, while the clusters appear to have been assembled mostly above $z=1$ and the bulk of the hot gas to be present already during the earliest phases of the cluster evolution, the heavy metal content was progressively released to the plasma by the stellar evolution during the whole Hubble time, as anticipated in Sect. 4.2.4.